Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – Nov/Dec – 2017**

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| **Code :** | **15MA3012** | **Duration :** | **3hrs** |
| **Sub. Name :** | **FUNCTIONAL ANALYSIS** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | Let *p* be a real number such that . Prove that  is a Banach Space. | CO1 | 10 |
| b. | Let *X, Y*be vector space, both real or both complex. Let be a linear operator with domain  and range . Then prove that  a) The inverse  exists if and only if *Tx = 0 x = 0.*  b) If  exists, it is a linear operator. | CO1 | 10 |
| (OR) | | | | |
| 2. | a. | If *Y* is a Banach Space, then prove that *B(X, Y)* is a Banach Space | CO1 | 10 |
| b. | Let , , ….. be n normed spaces. Then prove that  is a normed space under for . | CO1 | 10 |
|  |  |  |  |  |
| 3. | a. | Prove dual space of is . | CO2 | 12 |
|  | b. | Let *M* is a closed linearsubspace of a normed linear space *N* and let is a vector not in *M*, then prove there exists a functional  in  such that | CO2 | 8 |
| (OR) | | | | |
| 4. |  | State and prove Uniform Boundedness Theorem. | CO2 | 20 |
|  |  |  |  |  |
| 5. |  | Prove that a bounded linear operator *T* from a Banach Space *X* onto Banach Space *Y* is an open mapping. | CO2 | 20 |
| (OR) | | | | |
| 6. | a. | State and Prove Closed Graph theorem. | CO2 | 10 |
|  | b. | Discuss the solution of linear equation using fixed point theorem. | CO2 | 10 |
|  |  |  |  |  |
| 7. | a. | Prove that A normed space is an inner- product space if and only if the norm of the normed space satisfies the parallelogram law. | CO3 | 14 |
|  | b. | Prove that  of a closed subspace *Y* of a Hilbert Space *H* is a null space *N(P)* of the orthogonal projection *P* of *H* onto *Y*. | CO3 | 6 |
| (OR) | | | | |
| 8. | a. | State and Prove Gram-Schmit Orthogonalization. | CO3 | 10 |
|  | b. | For any subset  of Hilbert Space *H*, show that the Span of *M* is dense in *H* if and only if . | CO3 | 10 |
|  | | **Compulsory:** |  |  |
| 9. | a. | Prove that every bounded linear functional on Hilbert space then there exists a unique vector  such that . | CO3 | 8 |
|  | b. | If  and  are normal operators on *H* with the property that either commutes with adjoint of the other, then prove that  and  are normal | CO3 | 7 |
|  | c. | Prove that a linear operator *T* on Hilbert Space *H* is Unitary if and only if adjoint of *T* exists and . | CO3 | 5 |

ALL THE BEST